

TORSION OF SHAFTS & SPRINGS

Torsion:-

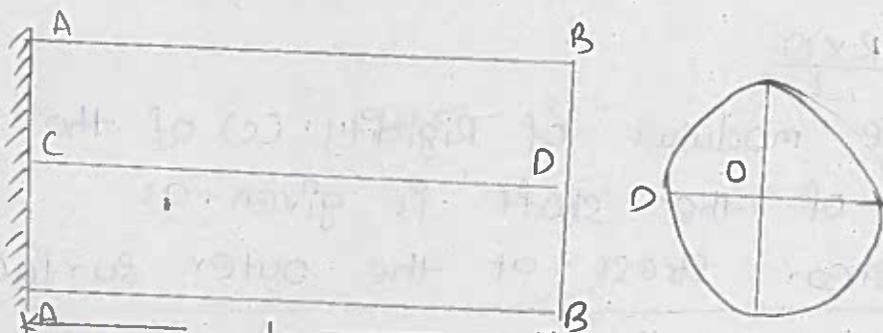
[Rebit strength of material - Dr. R.K. Bandal]

A shaft said to be in torsion, when equal and opposite torques are applied at the two ends of the shaft. A torque is equal to the product of force applied (tangentially to the ends of a shaft) and radius of the shaft.

Derivation of a shear stress produced in a circular shaft subjected to torsion:-

when a circular shaft is subjected to torsion, shear stresses are setup in the material of the shaft. to determine the magnitude of the shaft. to determine shear stresses at any point on the shaft,

consider a shaft fixed at one end A-A and free at the end B-B as shown in fig ①. Let CD is any line on the outer surface of the shaft



$R$  = Radius of shaft

$L$  = length of shaft

$T$  = Torque applied at the end beam B-B.

$\tau$  = shear stress induced at the surface of the shaft due to the torque.

$C$  = modulus of rigidity of the material of shaft.

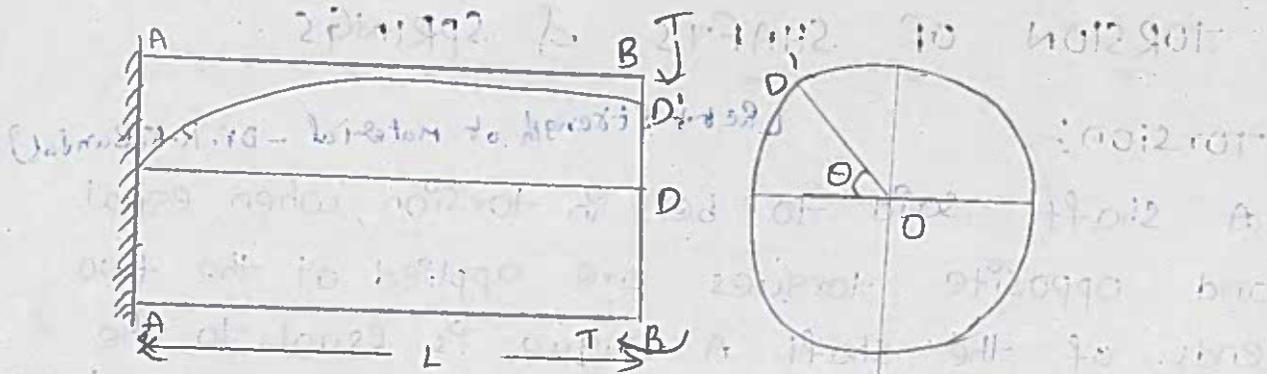
$\phi$  =  $\angle DCD'$  also equal to shear strain

$\theta$  =  $\angle DOD'$  and also called angle of twist.

The point D will be shifted to D' and hence line CD will be deflected CD' as shown in fig ②

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The line OD will be shifted to OD'



Now distortion at the outer surface due to torque (T) = DD'

$$\therefore \text{Shear strain at outer surface} = \frac{\text{Distortion at the outer surface}}{\text{length of shaft}} = \frac{DD'}{L}$$

$$= \frac{DD'}{OD} = \tan \phi \quad (\phi \text{ is small s.t. } \tan \phi = \phi)$$

$\therefore$  Shear strain at outer surface

$$\phi = \frac{DD'}{L} \quad \text{--- (1)}$$

Now from fig (2) to consider the

$$\text{arc } DD' = OD \times \theta = R\theta \quad [OD = R = \text{radius of shaft}]$$

Sub the value of DD' in equ (1), we get shear strain at outer surface

$$\phi = \frac{R \times \theta}{L}$$

Now the modulus of rigidity (C) of the material of the shaft is given as

$$C = \frac{\text{shear stress at the outer surface}}{\text{shear strain at the outer surface}}$$

$$C = \frac{\tau}{\frac{R\theta}{L}} = \frac{\tau \times L}{R\theta}$$

$$\frac{C\theta}{L} = \frac{\tau}{R}$$

$$\tau = \frac{R \times C \times \theta}{L}$$

Now for a given shaft subjected to given torque T, the value of C, theta and L, are the constant. Hence, shear stress produced is proportional to the radius (R)

$$\tau \propto R$$

$$\frac{T}{R} = \text{constant} \quad \text{--- (1)}$$

If  $q$  is the shear stress induced at a radius ( $R$ ) from the centre of the shaft then

$$\frac{T}{R} = \frac{q}{r} \quad \text{--- (1)}$$

$$\frac{T}{R} = \frac{C\theta}{L}$$

$$\frac{T}{R} = \frac{C\theta}{L} = \frac{q}{r}$$

If the  $f_s$  is the intensity of shear stress on any layer at a distance  $r$  from the centre of the shaft, then

$$\frac{f_s}{r} = \frac{f_s}{R} = \frac{C\theta}{L} \quad \text{--- (2)}$$

Consider an elementary ring area of radius  $r$  and an area  $da$  on the c/s of shaft.

Shear force offered by elemental area

$$i.e., df = f_s' \times \frac{r}{R} da$$

Moment of resistance offered by

$$dm = \text{force} \times \text{distance}$$

$$dm = f_s' \times \frac{r}{R} da \times r = \frac{f_s'}{R} r^2 da$$

Total resistance offered by whole shaft

$$T = \int \frac{f_s}{R} r^2 da$$

$$= \frac{f_s}{R} \int r^2 da = \frac{f_s}{R} J$$

$$(\because \int r^2 da = J)$$

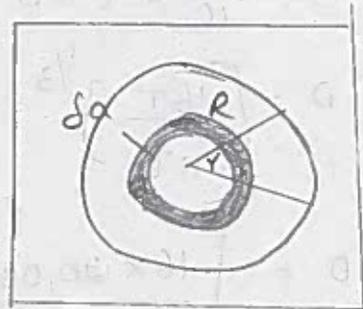
$$\frac{T}{J} = \frac{f_s}{R}$$

By equating equation

$$\frac{T}{J} = \frac{f_s}{R} = \frac{C\theta}{L}$$

(or)

$$\frac{T}{J} = \frac{T}{R} = \frac{C\theta}{L}$$



c/s of shaft

Maximum torque transmitted by a circular

solid shaft:-

$$T = \frac{\pi}{16} \tau D^3$$

- 1) A solid shaft of 150 mm dia is used to transmit torque. Find the max. torque transmitted by the shaft if the max. shear stress induced to the 45 N/mm<sup>2</sup>.

G.D:-

$$D = 150 \text{ mm}$$

$$\tau = 45 \text{ N/mm}^2$$

$$T = \frac{\pi}{16} \tau D^3$$

$$= \frac{\pi}{16} \times 45 \times 150^3$$

$$T = 29820586.52 \text{ N/mm}$$

- 2) The shearing stress of the solid shaft is not to exceed 40 N/mm<sup>2</sup>. when torque is transmitted 2000 N/m. Determine the min dia of the shaft

G.D:-

$$\tau = 40 \text{ N/mm}^2$$

$$T = 20,000 \text{ N/m} = 20000 \times 10^3 \text{ N.mm}$$

$$D = ?$$

$$T = \frac{\pi}{16} \tau D^3$$

$$D = \left[ \frac{16T}{\pi \tau} \right]^{1/3}$$

$$D = \left[ \frac{16 \times 20,000 \times 10^3}{\pi \times 40} \right]^{1/3}$$

$$D = 136.5 \text{ mm}$$

Torque transmitted by a hollow circular shaft:

$$T = \frac{\pi}{16} \tau \left[ \frac{D_o^4 - D_i^4}{D_o} \right]$$

Power transmitted by a shaft:

$$P = \frac{2\pi NT}{60} \text{ watts}$$

$$= \omega \times T \quad \left[ \because \frac{2\pi N}{60} = \omega \right]$$

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1) In a hollow circular shaft of outer and inner dia 20cm and 10cm respectively, the shear stress is not exceed 40 N/mm<sup>2</sup>. Find max. torque which the shaft can be easily transmitted.

$$\text{Internal dia } (D_i) = 10 \text{ cm} = 100 \text{ mm}$$

$$\text{External dia } (D_o) = 20 \text{ cm} = 200 \text{ mm}$$

$$\tau = 40 \text{ N/mm}^2$$

$$T = \frac{\pi}{16} \tau \left[ \frac{D_o^4 - D_i^4}{D_o} \right]$$

$$= \frac{\pi}{16} \times 40 \left[ \frac{200^4 - 100^4}{200} \right]$$

$$= 589048.62 \text{ N/mm}$$

2) A hollow shaft of external dia 120 mm transmits 300 kW power at 200 r.p.m. Determine the max. internal dia if the max. stress of the shaft is not exceed 60 N/mm<sup>2</sup>.

$$\text{External dia } (D_o) = 120 \text{ mm}$$

$$P = 300 \text{ kW} = 300 \times 10^3 \text{ watts}$$

$$N = 200 \text{ r.p.m}$$

$$\text{Internal dia } (D_i) = ?$$

$$\tau = 60 \text{ N/mm}^2$$

Power:-

$$P = \frac{2\pi NT}{60}$$

$$T = \frac{P \times 60}{2\pi \times N} = \frac{300 \times 10^3 \times 60}{2 \times \pi \times 200}$$

$$T = 14323.9 \times 10^3 \text{ N/mm}$$

$$T = \frac{\pi}{16} \tau \left[ \frac{D_o^4 - D_i^4}{D_o} \right]$$

$$14323.9 \times 10^3 = \frac{\pi}{16} \times 60 \left[ \frac{120^4 - D_i^4}{120} \right]$$

$$\boxed{D_i = 88.54 \text{ mm}}$$

3) A solid steel shaft is to transmit 75 kW at 200 r.p.m taking allowable shear stress as  $17 \text{ N/mm}^2$  find suitable dia for the shaft if the max. torque transmitted each revolution exceeds the mean by 30%.

$$P = 75 \text{ kW} = 75 \times 10^3 \text{ W}$$

$$N = 200 \text{ r.p.m}$$

$$\tau = 17 \text{ N/mm}^2$$

$$T = ?$$

$$P = \frac{2\pi NT}{60} \omega$$

$$75 \times 10^3 = \frac{2\pi \times 200 \times T}{60}$$

$$T = 3580.98 \text{ N.m} = 3580980 \text{ N.mm}$$

$$T_{\text{max}} = 1.3T \text{ (30% of revolution)}$$

$$= 100\% + 30\%$$

$$= \frac{130}{100} = 1.3$$

$$T_{\text{max}} = 1.3T$$

$$= 1.3T \times 3580980$$

$$= 4655274 \text{ N.mm}$$

Max. torque transmitted by a solid shaft

$$\tau_{\max} = \frac{9\pi D^3}{16} \tau$$

$$4655274 = \frac{\pi}{16} \times 70 \times D^3$$

$$D = 69.7063 \text{ mm}$$

Polar moment of Inertia

$$J = \frac{\pi}{32} D^4$$

Polar modulus:-

Polar modulus is defined as the ratio of the polar moment of the inertia to the radius of the shaft. It is also called as torsional section modulus.

→ It is denoted by  $Z_p$

$$Z_p = \frac{J}{R}$$

a) solid shaft  $(Z_p) = \frac{\pi}{16} D^3 = \frac{\pi D^4}{32 \times 16} \times \frac{3}{D/2} = \frac{\pi}{16} D^3$

b) for hollow shaft:-

$$Z_p = \frac{\pi}{16} [D_o^4 - D_i^4]$$

Strength of the shaft:-

The strength of the shaft means the max. torque (or) max. power the shaft can be transmit.

torsional rigidity (or) stiffness of the shaft:-

Torsional rigidity is defined as the product of modulus of rigidity and polar moment of inertia of the shaft.

$$\text{torsional rigidity} = C \times J$$

1) Determine the dia of a solid steel which will transmit 90 kW at 160 r.p.m also determine the length of the shaft if the twist must not exceed  $1^\circ$  over the entire length. The max. shear stress is limited to  $60 \text{ N/mm}^2$ . Take the value of modulus of rigidity  $8 \times 10^4 \text{ N/mm}^2$ .

G.D:-

$$P = 90 \text{ kW} = 90 \times 10^3 \text{ W}$$

$$N = 160 \text{ r.p.m}$$

$$\tau = 60 \text{ N/mm}^2$$

$$C = 8 \times 10^4 \text{ N/mm}^2$$

$$\theta = 1^\circ = \frac{\pi}{180} \text{ radians}$$

$$P = \frac{2\pi NT}{60}$$

$$90 \times 10^3 = \frac{2 \times \pi \times 160 \times T}{60}$$

$$T = 5371.479 \text{ N.m}$$

$$= 5371.479 \times 10^3 \text{ N.mm}$$

$$T = \frac{\pi}{16} \times \tau \times D^3$$

$$5371.479 \times 10^3 = \frac{\pi}{16} \times 60 \times D^3$$

$$D = 76.96 \text{ mm} \approx 77 \text{ mm}$$

$$\frac{T}{J} = \frac{C\theta}{L} = \frac{\tau}{r}$$

$$\frac{T}{R} = \frac{C\theta}{L}$$

$$\frac{60}{38.5} = \frac{8 \times 10^4 \times \frac{\pi}{180}}{L}$$

$$L = 895.935 \text{ mm} \approx 896 \text{ mm}$$

2) Determine the dia of a solid shaft which will transmit 300 kw at 250 r.p.m. the max. shear stress not to exceed 30 N/mm<sup>2</sup> and twist should not be more than 1° in a shaft length of 2m. take modulus of rigidity  $1 \times 10^5$  N/mm<sup>2</sup>.

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$$P = 300 \text{ kw} = 300 \times 10^3 \text{ w}$$

$$N = 250 \text{ r.p.m}$$

$$\tau = 30 \text{ N/mm}^2$$

$$\theta = 1^\circ = \frac{\pi}{180} \text{ radians}$$

$$L = 2 \text{ m} = 2000 \text{ mm}$$

$$C = 1 \times 10^5 \text{ N/mm}^2$$

$$P = \frac{2\pi NT}{60}$$

$$300 \times 10^3 = \frac{2 \times \pi \times 250 \times T}{60}$$

$$T = 11459.155 \text{ N.m}$$

$$= 11459.155 \times 10^3 \text{ N.mm}$$

$$T = \frac{\pi}{16} \times \tau \times D^3$$

$$11459.155 \times 10^3 = \frac{\pi}{16} \times 30 \times D^3$$

$$D = 124.834 \text{ mm} \approx 125 \text{ mm}$$

(ii) Dia of the shaft when twist should not be more than 1°

$$\frac{T}{J} = \frac{C\theta}{L}$$

$$\frac{11459.1 \times 10^3}{\frac{\pi}{32} D^4} = \frac{1 \times 10^5 \times 6.017}{2000}$$

$$D = 107.541 \text{ mm} \approx 108 \text{ mm}$$

The suitable dia of shaft is the greater of the two values

∴ the dia of the shaft

$$= 125 \text{ mm (nearly)}$$

If the dia. is taking smaller of the values

say,  $107.5 \text{ mm}$  then from the eqn

$T = \frac{\pi}{16} \times \tau \times d^3$  the value of shear

stress  $\tau$

$$T = \frac{\pi}{16} \times \tau \times d^3$$

$$11459.15 \times 10^3 = \frac{\pi}{16} \times \tau \times 107.5^3$$

$$\tau = 46.978 \text{ N/mm}^2$$

- 3) A hollow shaft dia ratio  $3/8$  (internal dia to outer dia) is to transmit  $375 \text{ kW}$  power at  $100 \text{ r.p.m}$ . the max. torque being  $20\%$  greater than the mean the shear stress is not exceed  $60 \text{ N/mm}^2$  and twisting a length of  $4 \text{ m}$ . not exceed  $2^\circ$ . Calculate its internal and external dia which would satisfy both the above conditions. Assume modulus of rigidity  $C = 0.85 \times 10^5 \text{ N/mm}^2$ .

$$\text{Diameter ratio } \frac{D_i}{D_o} = \frac{3}{8}$$

$$D_i = \frac{3}{8} D_o$$

$$P = 375 \text{ kW} = 375 \times 10^3 \text{ W}$$

$$N = 100 \text{ r.p.m}$$

$$\text{Max. torque } (T_{\text{max}}) = 1.2 T_{\text{mean}}$$

$$\text{Max. twist } \theta = 2^\circ = 2 \times \frac{\pi}{180} = 0.034 \text{ radians}$$

$$\text{Modulus of rigidity } C = 0.85 \times 10^5 \text{ N/mm}^2$$

$$P = 2\pi NT$$

$$375 \times 10^3 = \frac{2 \times \pi \times 100 \times T}{60}$$

$$T_{\text{mean}} = 35809.8 \text{ N.m}$$

$$T_{\text{mean}} = 35809.8 \times 10^3 \text{ N.m} \quad \text{unit-1 Pg. no-10/22}$$

$$T_{\max} = 1.2 \times 35809.8 \times 10^3$$

$$T_{\max} = 42972000 \text{ N}\cdot\text{mm}$$

1) Diameter of the shaft when shear stress is not exceed  $60 \text{ N/mm}^2$ .

$$T_{\max} = \frac{\pi}{16} \tau \left[ \frac{D_o^4 - D_i^4}{D_o} \right]$$

$$42972000 = \frac{\pi}{16} \times 60 \times \left[ \frac{D_o^4 - \left(\frac{3}{8} D_o\right)^4}{D_o} \right]$$

$$D_o = 154.97 \approx 155 \text{ mm}$$

$$D_i = \frac{3}{8} D_o$$

$$= \frac{3}{8} \times 154.97$$

$$= 58.11 \text{ mm}$$

2) Diameter of the shaft when the twist is not exceed  $2^\circ$ .

$$\frac{T}{J} = \frac{C \times \theta}{L}$$

$$\frac{42972000}{\frac{\pi}{32} \left[ D_o^4 - \left(\frac{3}{8} D_o\right)^4 \right]} = \frac{0.85 \times 10^5 \times 0.034}{4000}$$

$$D_o = 157.6 \text{ mm}$$

$$D_i = \frac{3}{8} D_o$$

$$= \frac{3}{8} \times 157.6 = 59.1 \text{ mm}$$

4) A solid circular shaft transmits 75 kW power at 200 r.p.m. Calculate the shaft dia, if the twist in the shaft is not exceed  $1^\circ$  in 2m length of shaft, shear stress limited to  $50 \text{ N/mm}^2$ ,  $C = 1 \times 10^5 \text{ N/mm}^2$ .

$$P = 75 \text{ kW} = 75 \times 10^3 \text{ W}$$

$$N = 200 \text{ r.p.m}$$

$$d = ?$$

$$\theta = 1^\circ = \frac{\pi}{180} \text{ radians}$$

$$l = 2 \text{ m} = 2000 \text{ mm}$$

$$\tau = 50 \text{ N/mm}^2$$

$$C = 1 \times 10^5 \text{ N/mm}^2$$

$$P = \frac{2\pi NT}{60}$$

$$75 \times 10^3 = \frac{2 \times \pi \times 200 \times T}{60}$$

$$T = 3580.986 \text{ N.m}$$

$$= 3580.986 \times 10^3 \text{ N.mm}$$

$$T = \frac{\pi}{16} \times \tau \times D^3$$

$$3580.986 \times 10^3 = \frac{\pi}{16} \times 50 \times D^3$$

$$D = 71.44$$

Diameter of the shaft when the twist in the shaft not to exceed  $1^\circ$ .

$$\frac{T}{J} = \frac{C\theta}{L}$$

$$\frac{3580.986 \times 10^3}{\frac{\pi}{32} D^4} = \frac{1 \times 10^5 \times \frac{\pi}{180}}{2000}$$

$$D = 80.4 \approx 81 \text{ mm}$$

Suitable diameter from the two values to choose the greater value.

# SPRINGS

Springs are the elastic bodies which observe energy due to resilience. The observe energy may be released as and when required.

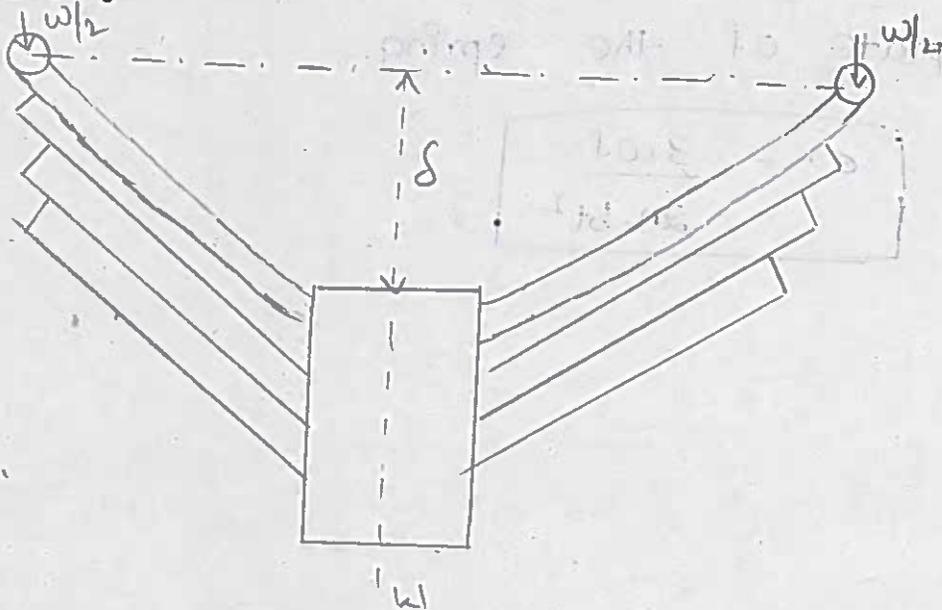
## Classification of Springs

- 1) Leaf spring (or) Laminated spring.
- 2) Helical spring.

### 1) Leaf spring:-

The laminated springs are used to observe shocks in railways wagons, coaches and road vehicles, lorries, tractors.

Expression for max. bending stress developed in plate:-



where  $w$  = point load acting at the centre of the lower most plate, will be shared equally on the 2 ends of the top plane as shown in fig.

∴ B.M at the centre = load at one end  $\times l/2$

$$M = \frac{w}{2} \times l/2 = \frac{wl}{4} \quad \text{--- (1)}$$

The M.I of each plate  $I = \frac{bt^3}{12}$

But the relation among bending stress, B.M, M.I is given by

$$\frac{M}{I} = \frac{\sigma}{y} \quad (\text{B.M equation})$$

$$M = \frac{\sigma}{y} \times I = \frac{\sigma \times \frac{bt^3}{12}}{t/2} = \frac{\sigma \times bt^2}{6}$$

∴ Total Resisting moment by n plates

$$= n \times m = \frac{n \times \sigma bt^2}{6} \quad \text{--- (2)}$$

Equating (1) & (2)

$$\frac{wl}{4} = \frac{n \times \sigma bt^2}{6}$$

$$\sigma = \frac{3wl}{4nbt^2} = \frac{3wl}{2n \cdot bt^2}$$

The above the equation gives the max. shear stress developed in the plate of the spring.

$$\sigma = \frac{3wl}{2n \cdot bt^2}$$

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Expression for central deflection of the

leaf spring:-

Now  $R$  = Radius of the plate to which they are bend.

From the  $\Delta ACO$ .

$$AO^2 = AC^2 + CO^2$$

$$R^2 = \left(\frac{d}{2}\right)^2 + (R - \delta)^2$$

$$= \frac{d^2}{4} + R^2 + \delta^2 - 2R\delta$$

$$R^2 = \frac{d^2}{4} + R^2 - 2R\delta \quad (\delta \text{ is small quantity})$$

$$2R\delta = \frac{d^2}{4}$$

$$\delta = \frac{d^2}{4 \times 2R} = \frac{d^2}{8R} \quad \text{--- (1)}$$

But the relation between bending stress, modulus of elasticity and radius of curvature is given by

$$\frac{\sigma}{y} = \frac{E}{R} \quad [\text{Bending equation}]$$

$$R = \frac{E \times y}{\sigma}$$

$$= \frac{E \times t}{2\sigma} \quad (y = t/2)$$

Sub the  $R$  value from equ (1)  $\therefore$

$$\delta = \frac{d^2}{8 \times \frac{E \times t}{2\sigma}} = \frac{d^2 \times 2\sigma}{8Et} = \frac{\sigma d^2}{4Et}$$

$$\boxed{\delta = \frac{\sigma d^2}{4Et}}$$

The above equation gives the central deflection of spring.

1) A leaf spring carries a central load of 3000 N. This leaf spring is to be made of 10 steel plates 5 cm wide and 6 mm thick. If the bending stress is limited to 150 N/mm<sup>2</sup>. Determine the

- 1) length of the spring
- 2) deflection at the centre of the spring.
- 3) take  $E = 2 \times 10^5$  N/mm<sup>2</sup>

NO. of (n) = 10 steel plates

$$W = 3000 \text{ N}$$

$$t = 6 \text{ mm}$$

$$\text{width (b)} = 5 \text{ cm} = 50 \text{ mm}$$

$$\sigma = 150 \text{ N/mm}^2$$

$$E = 2 \times 10^5 \text{ N/mm}^2$$

$$\sigma = \frac{3Wl}{2nbt^2}$$

$$l = \frac{\sigma 2nbt^2}{3W}$$

$$l = \frac{150 \times 2 \times 10 \times 50 \times 6^2}{3 \times 3000}$$

$$\therefore \boxed{l = 600 \text{ mm}}$$

$$\delta = \frac{\sigma l^2}{4Et} = \frac{150 \times 600^2}{4 \times 2 \times 10^5 \times 6}$$

$$\therefore \boxed{\delta = 11.25 \text{ mm}}$$

Q2

A laminated spring 1m length is made up of plates each 5cm wide and 1cm thick. If the bending stress for the plate is limited to 100 N/mm<sup>2</sup>. How many plates could be required to enable the springs to carry a central point load of 2kN. What is the deflection under the load.

$$E = 2.1 \times 10^5 \text{ N/mm}^2$$

$$L = 1\text{m} = 1000 \text{ mm}$$

$$b = 5\text{cm} = 50 \text{ mm}$$

$$t = 1\text{cm} = 10 \text{ mm}$$

$$\sigma = 100 \text{ N/mm}^2$$

$$W = 2\text{kN} = 2 \times 10^3 \text{ N}$$

$$\sigma = \frac{3Wl}{2n \cdot bt^2}$$

$$100 = \frac{3 \times 2 \times 10^3 \times 1000}{2 \times n \times 50 \times 10^2}$$

$$n = 6$$

$$\delta = \frac{\sigma \cdot l^2}{4Et}$$

$$\delta = \frac{100 \times 1000^2}{4 \times 2.1 \times 10^5 \times 10}$$

$$\delta = 11.904 \text{ mm}$$

### Helical springs :-

Helical springs are the thick springs wires coiled into a helix.

1) closed coiled helical spring

2) open coiled helical spring

1) closed coiled helical spring :-

closed coiled helical springs in which helix angle is very small ( $\alpha \approx 0$ ) in other words the pitch between two adjacent turns is small

### Expression for max. shear stress induced in wire :-

To consider the closed coil helical spring subjected to an axial load as shown in above the fig...

Now twisting moment on the wire

$$T = W \times R \quad \text{--- (1)}$$

But twisting moment also given by

$$T = \frac{\pi}{16} \tau d^3 \quad \text{--- (2)}$$

to equating the (1) & (2)

$$W \times R = \frac{\pi}{16} \tau d^3$$

$$\tau = \frac{16 W \times R}{\pi \times d^3}$$

The above the equation gives the max.

shear stress induced in the wire. unit - 1 Pg. no - 18/22

Expression for deflection of spring

total length of the wire = length of  
1 coil x no. of coil

$$l = 2\pi R \times n$$

Strain energy stored by the spring

$$U = \frac{\tau^2}{4c} \times \text{volume}$$

$$= \left( \frac{16wR}{\pi d^3} \right)^2 \times \frac{1}{4c} \times \left( \frac{\pi}{4} d^2 \times 2\pi R \times n \right)$$

$$= \frac{16}{256 \times 4} \times \left( \frac{w^2 R^2}{d^6} \right) \times d^2 \times \frac{\pi R \times n \times \pi}{c}$$

$$U = \frac{32 w^2 R^2}{c d^4} \times n$$

$$U = \frac{32 w^2 R^3 n}{c d^4} \quad \text{--- (1)}$$

work done on spring = Average load x Deflection

$$= \frac{1}{2} w \times \delta \quad \text{--- (2)}$$

Equating (1) and (2)

$$\frac{32 w^2 R^3 n}{c d^4} = \frac{1}{2} w \times \delta$$

$$\delta = \frac{32 w^2 R^3 n \times 2}{w \times c d^4}$$

$$\delta = \frac{64 w R^3 n}{c d^4}$$

expression for stiffness spring:-

the stiffness of spring  $S = \frac{\text{load}}{\text{unit deflection}}$

$$= \frac{W}{\delta}$$

- 1) A closed coiled helical spring of 10cm mean dia upto to 1cm dia and 20 turns the spring carries a axial load of 200N. Determine the shearing stress taking the value of  $C = 8.4 \times 10^4 \text{ N/mm}^2$ . Determine the deflection when carrying this load and also calculate the stiffness of the spring and frequency of free vibration for a mass hanging from it.

$$\text{Dia } (d) = 1 \text{ cm} = 10 \text{ mm}$$

$$\text{Mean dia } (D) = 10 \text{ cm} = 100 \text{ mm}$$

$$\text{Load } (W) = 200 \text{ N}$$

$$R = \frac{100}{2} = 50 \text{ mm}$$

$$\text{No. of turns } (n) = 20$$

$$C = 8.4 \times 10^4 \text{ N/mm}^2$$

1) Deflection of the spring

$$\delta = \frac{64 W R^3 n}{C d^4}$$

$$= \frac{64 \times 200 \times 50^3 \times 20}{8.4 \times 10^4 \times 10^4}$$

$$\delta = 38.095 \text{ mm}$$

2) Max. shear stress

$$\tau = \frac{16 W R}{\pi d^3}$$

$$\tau = \frac{16 \times 200 \times 50}{\pi \times 10^3} = 50.929 \text{ N/mm}^2$$

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3) stiffness: (1)

$$S = \frac{Cd^4}{64R^3n}$$
$$= \frac{8.4 \times 10^4 \times 10^4}{64 \times 50^3 \times 20}$$

$$S = 5.25 \text{ N/mm}$$

4) Frequency

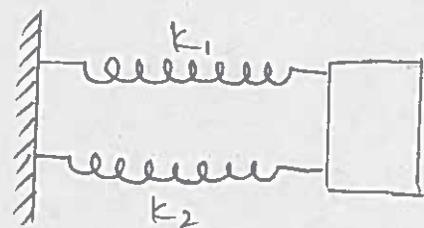
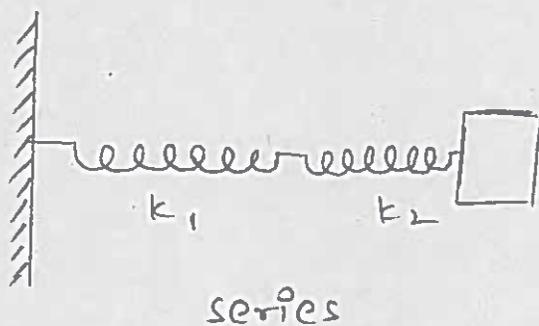
$$T = \frac{1}{2\pi} \sqrt{\frac{g}{\delta}}$$
$$= \frac{1}{2\pi} \sqrt{\frac{98.10}{38.09}}$$

$$T = 2.55 \text{ cycle/sec}$$

### Series and parallel springs

In mechanics, two or more springs are said to be in series when they are connected end-to-end or point to point, and it is said to be in parallel.

when they are connected side-by-side, in both cases, so as to act as a single spring.



More generally, two or more springs are in series when any external stress applied to the ensemble gets applied to each spring without change of magnitude, and the amount strain (deformation) of the ensemble is the sum of the strains of individual springs. Conversely, they are said to be in parallel if the strain of the ensemble is their common strain, and the stress of the ensemble is the sum of their stresses.

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$$F = \sum F_i$$

Two or more springs are in parallel if the strain of the ensemble is their common strain, and the stress of the ensemble is the sum of their stresses.

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